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Comment on ''Models of intermediate spectral statistics''

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In this Comment we point out that the semi-Poisson is well suited only as a reference point for the so-called ''intermediate statistics,'' which cannot be interpreted as a universal ensemble, like the Gaussian orthogonal ensemble or the Poissonian statistics. In Ref. [2] it was proposed that the nearest-neighbor distribution $P(s)$ of the spectrum of a Poissonian distributed matrix perturbed by a rank one matrix is similar to the semi-Poisson distribution. We show, however, that the $P(s)$ of this model differs considerably in many aspects from the semi-Poisson. In addition, we give an asymptotic formula for $P(s)$ as $s\to 0$, which gives $P'(0)=\pi\sqrt{3}/2$ for the slope at $s=0$. This is different not only from the GOE case, but also from the semi-Poisson prediction.

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The motivation for this Comment stems from our impression that the article, ''Models of intermediate spectral statistics,'' can easily be misinterpreted in two ways: One might be led to believe that (i) the semi-Poisson distribution is universal, and (ii) the universality class of ''intermediate statistics'' is as well defined and established as, for example, the Poisson ensemble or the Gaussian orthogonal ensemble (GOE). In this Comment, we will argue that both statements are wrong.

Shklovskii *et al.* [1] introduced a numerical spacing distribution $P_T(s)$, and conjecture that it is universal for some class of disordered systems at the metal-insulator transition point. In Refs. [2,3] random matrix models have been proposed in order to reproduce these findings. Particularly, in Ref. [2] various models are presented, and with reference to [1] considered as members of a ''third universal ensemble'' of systems showing so-called intermediate statistics. The Poissonian and the Gaussian ensemble (for clarity, consider orthogonal ensembles only) are considered as the two primary universality classes in this list.

As in the Poissonian and in the GOE case, where the respective members have common and unique statistical properties, one would expect the same to hold for the models showing intermediate statistics. In Ref. [2] the authors concentrate on the distribution of nearest-neighbor spacings. In the Poissonian case it is given by $P(s)=\exp(-s)$, and in the GOE case it is close to the well-known Wigner surmise $P(s)\approx(\pi s/2)\exp(-\pi s^2/4)$. In the case of the intermediate statistics, the candidate proposed in Ref. [2] is the semi-Poisson distribution $P(s)=4s\exp(-2s)$.

One of the examples showing intermediate statistics, presented in Ref. [2], is defined as

$$H_{mn}=e_n\delta_{mn}+t_m t_n . \qquad (1)$$

H is a $N\times N$ matrix, e_n are mutually independent random variables uniformly distributed over a finite interval, and the t_n are chosen with equal absolute value squared $t_n^2=r$.

Due to the discussion in Ref. [2], an unprejudiced reader might believe that the correlation properties of the matrix model are similar to semi-Poisson. In what follows we will show that the level spacing distribution in the case of this model is in fact very different from the proposed semi-Poisson distribution. This discrepancy can already be seen in a figure published in Ref. [4], but the problem is not discussed there.

In Fig. 1 we present the numerical result for $P(s)$ obtained for an ensemble of 1000 matrices of dimension 750. For the statistical analysis we only used one third of the states in the center of the spectral region. The numbers e_n are uniformly distributed over an interval $[-1, 1]$ and the elements of the vector $\vec{t}$ are chosen as $t_i=\sqrt{\alpha/(\pi\rho N)}$, where ρ is the level density in the center of the spectrum, N is the dimension of the matrix, and $\alpha=10$ is the coupling constant. (We checked that a larger coupling does not change the numerical results). Figure 1(a) demonstrates the qualitative differences in the behavior of $P(s)$ between the random matrix model, the semi-Poisson distribution, and the exact GOE. For values of $0<s\lesssim 2.5$, it is impossible to decide whether the numerical data lie closer to any one of the two reference curves. In both cases, the deviations are well above the statistical error. Even though the numerical data show a linear increase at small s and an approximately exponential falloff at large s, the spacing distribution of the matrix model is not close to semi-Poisson.

Figure 1(b) shows a magnification of the interval $0\leqslant s\leqslant 1/2$ using the same data as in Fig. 1(a). Here we additionally plotted the asymptotic result (3) for the present model as a dotted line. The basic idea for the derivation of Eq. (3) is the following. In order to get a short distance between two neighbored levels in the spectrum of H, three eigenvalues of

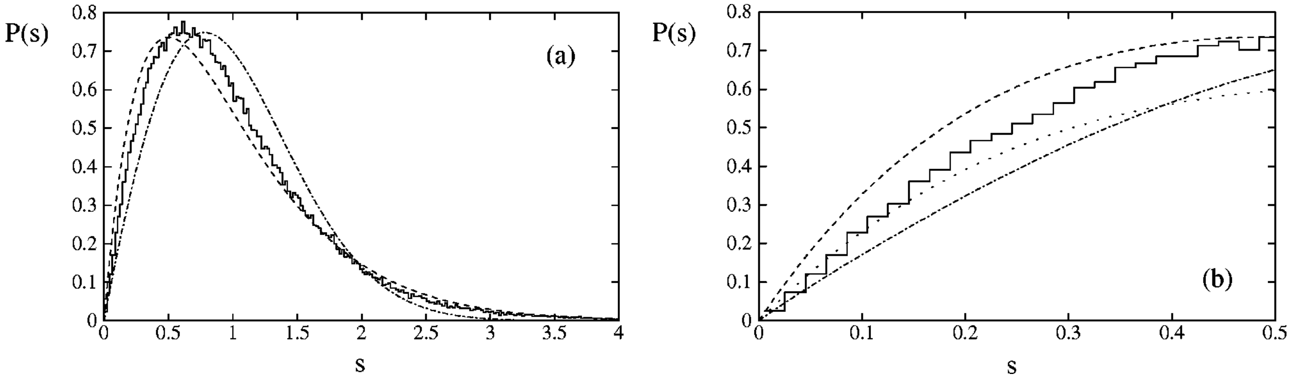

FIG. 1. (a) Nearest-neighbor distribution $P(s)$ for the model (1) compared with the semi-Poisson distribution (dashed line) and the exact GOE (dashed-dotted). The vertical scale gives the probability of finding two adjacent levels at distance s. s is given in units of the unfolded average level spacing. (b) The same as in (a) for short distances. In addition, the theoretical result (3) is drawn as a dotted line.

H_0 have to come close together. Then the remaining levels, being relatively far away, can be neglected. Therefore, one can restrict the sum

$$K(E)=\sum_{i=1}^{N}\frac{t_i^2}{E-e_i}, \qquad (2)$$

whose roots define the eigenvalues of H, to those terms that contain the three consecutive eigenvalues. Resolving for the two roots, calculating their distance, and averaging over the levels e_i leads to the following formula

$$P(s)=\frac{9s}{4}\int_0^{\pi/2}d\phi \times\frac{\exp\left[-\frac{3s}{2}(\cos\phi+\sin\phi)/\sqrt{1+\sin(2\phi)/2}\right]}{1+\sin(2\phi)/2}. \qquad (3)$$

The dashed curve in Fig. 1(b) is obtained from a numerical integration of Eq. (3). At short distances, this approximation describes the numerical data much better than the semi-Poisson. A Taylor expansion of the integrand of Eq. (3) gives $P'(0)=\pi\sqrt{3}/2$ for the slope at $s=0$, the same result as found in Ref. [2].

A detailed numerical investigation of several statistical properties of this kind of model can be found also in Refs. [4–6].

To conclude, the models discussed in Ref. [2] have two common properties: (i) the linear increase of the nearest-neighbor distribution $P(s)$ at small s, and (ii) its approximate exponential falloff at large s. As a typical example we discussed the nearest-neighbor distribution of model (1). This suggests defining the class of systems showing intermediate statistics via those two properties only. Even though (i) and (ii) could be called universal for a large number of systems, this kind of universality is a rather weak one, in order to speak of the third universal ensemble. One should compare this to the Poisson or the GOE case, where the whole joint probability distribution (apart from the level density) is supposed to be shared by all the members of the respective classes.

In this context the semi-Poisson may serve as a reasonable reference point only. However, the comparison to the semi-Poisson is not an adequate procedure by which to decide whether a given system belongs to the class of intermediate statistics. Figure 1 clearly illustrates that even if huge discrepancies are present, the model in question may nevertheless belong to the class of intermediate statistics.

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